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Heat Transfer Analysis of the Ungrooved Disk of a Cooled, Multiplate Clutch

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A one-dimensional analysis is presented to determine the transient response of a disk due to a time varying heat flux boundary condition. The boundary condition alternates with a step change between a heating flux and a cooling flux which can occur over different time periods (simulating coolant grooves in the adjacent disk). For a multiplate clutch where the driving and reaction disks are of different materials, an approximation of the amount of the heat generated at the sliding interface which enters each disk is presented. Results are given to illustrate the effects of materials, heating and cooling fluxes, and time period on the transient temperature response. Increasing (reducing) the heating (cooling) flux, increasing (reducing) the time period of the heating (cooling) flux, decreasing the disk thickness or increasing the disk thermal diffusivity increases the disk temperature during the clutch engagement period.

Nomenclature

c = specific heat
d_0, d_m, e_m = variables in Fourier series approximation to the time-dependent heat flux boundary condition; Eqs. (15), (16), and (17), respectively
f = fraction of heat generation at interface which enters disk a
h = convection coefficient in coolant channel
I_0, I_n = integrals defined in Eq. (10)
i = integer from zero to some finite value used in boundary condition; Eq. (4)
k = thermal conductivity
L = disk thickness
m, n = indices of infinite series
P_N = normal pressure applied to rotating disks
q = heat flux
q_1, q_2 = heat fluxes in time-dependent boundary condition
r = radius or radial direction
s = dummy integration variable
T = temperature due to a constant boundary condition
T' = temperature due to a time varying boundary condition
T_f = fluid temperature
Th = temperature due to a constant boundary condition and an initial condition of zero temperature
Th' = temperature due to a time varying boundary condition and an initial condition of zero temperature
T_i = initial temperature distribution
$\bar{T}_i$ = uniform initial temperature
t = time
t_1 = time period used in periodic boundary condition
x = axial direction
α = thermal diffusivity, $k/(\rho c)$
β = function of time used in solution
$\Delta\phi$ = angular difference
θ = dimensionless temperature, $Th'/(q_1L/k) + d_0\alpha t/L^2$
λ_n = $(n\pi)/L$
μ_s = coefficient of sliding
ξ = function of x used in solution
π = numerical constant
ρ = density
τ = time period used in periodic boundary condition
Υ = torque per unit area
ϕ = angular direction
ψ = function of x and t used in solution
ω = difference between rotational speeds of two disks in sliding contact
ω_1 = angular speed of the reaction disk
ω_2 = angular speed of the driving disk

Subscripts
a = disk defined between $x = 0$ and $x = -L_a$
b = disk defined between $x = 0$ and $x = L_b$
i = initial value
0 = disk defined between $x = L$ and $x = 2L$

Introduction

CLUTCHES and brakes find use in varied applications of machinery, from automobiles and tanks to aircraft. As the torque that the device must transmit increases, the heat generated increases. Some applications require cooling to keep the device temperature below material limits or to shorten the allowable time period between cyclic use of the device. To this end, some means of cooling the clutch or brake plates either during or after the load is applied are necessary. Typically, these brake and clutch applications involve a rotating shaft, and correspondingly the device is geometrically in the shape of an annular disk. Cooling schemes range from surface grooved disks to hollow core disks through which some fluid (typically air or oil) is passed. If oil is used, the clutch temperature must be maintained below the temperature at which the oil begins to degrade.

To date, little analytical work has been done on intermittently heated and cooled plates in sliding contact (surface grooved disk), due to the complexity of the problem. One heat transfer model for a dry clutch has been given[1] where the energy flux decreases linearly with time, but only heating occurs during the cycle. Some empirical work has been done to determine suitable materials and configurations (e.g., num-

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ber and shape of grooves). A large portion of this type of work that has been done is company confidential. The situation described is part of a larger group of problems involving oscillatory boundary conditions and related literature is discussed in the following.

Irradiation of a solid by a pulsed laser has been modeled as a one-dimensional, transient problem in a semi-infinite solid due to a sinusoidal heat flux boundary condition with non-Fourier heat conduction.[2] Oscillatory sliding contacts have been investigated[3] to determine the tribological behavior of the two materials in dry contact. The interfacial heat flux is oscillatory due to the sliding velocity varying harmonically as one piece moves back and forth across the other. Quasisteady heat transfer across periodically contacting surfaces has been examined experimentally[4] and analytically.[5-8] Applications include meshing gear teeth, stamping processes, etc. The previous situations involve different boundary conditions, e.g., a heat flux which cycles in an on/off manner (between an arbitrary value and zero), or in a sinusoidal manner. The present work involves a boundary condition where the heat flux cycles between two arbitrary values with equal or different time periods. However, complete modeling of a reaction/driving disk pair to include even simple coolant channel geometries and wheels rotating at different speeds is a difficult task. This work addresses a simplified model of a periodically heated and cooled disk in sliding contact with another disk, and present results to quantify the response to different material properties, levels of heating and cooling fluxes, or time periods.

Analysis

For example, consider a clutch made up of several disks. The driving and reaction disks are alternated through the array. Assume the cooling fluid grooves are in both sides of the driving disk only, and that these grooves are simply radial grooves equally spaced around the circumference of the disk (Fig. 1a). If a frame of reference is chosen which rotates with the reaction disk (Fig. 1b), this effectively makes the reaction disk stationary and the driving disk move with some speed relative to the reaction disk (which is equal to the difference in speeds of the two disks with respect to a stationary frame of reference). At any point on the reaction disk, the boundary condition alternates between some heat generation (corresponding to the time the two disks are in contact at that point), and some cooling flux (due to a cooling groove passing over that point). The total time period for one cycle is the distance ($r\Delta\phi$) of one groove and one "land" (area where the wheels make contact) divided by the wheel rotational speed (ωr). This results in a time perid $\tau = \Delta\phi/\omega$. The time period t_1 accounts for the land only ($t_1 < \tau$), and is determined in a similar manner.

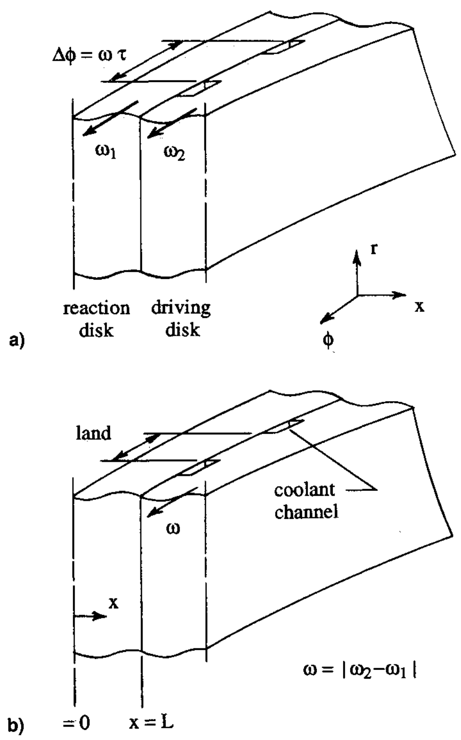

Fig. 1 Schematic of system illustrating half-thicknesses of two adjacent disks: a) stationary frame of reference and b) frame of reference rotating with reaction disk.

The work dissipated by the sliding disks is transformed into thermal energy at the contact interface which is transferred to both adjacent disks. The heat flux is determined as[9]

$$q = \mu_s P_N r\omega = \omega Y \tag{1}$$

If the disks are forced together under constant normal pressure, the initial difference in disk rotational speeds decreases until the two disks rotate together (typical of clutch operation, a brake decreases the speed of a rotating wheel). However, this results in an initial peak in the load on the disks. To optimize the load on the disks with respect to time, the pressure can be varied such that the load is more uniform over the engagement time (rectangular area under the load-time curve as opposed to some triangular shaped area). In other words, both the normal disk pressure and ω vary with time. This results in a nearly uniform heating flux due to the frictional effects of the applied load. Correspondingly, the intermittent heating flux at any given location (i.e., the heat flux at the surface alternates between a cooling and heating flux due to the oil grooved clutch surface), is taken as a constant value during the transient. If the heating flux cannot be taken as a constant, the solution can be broken up into steps with the heat flux (also cooling flux and time periods) constant over each step, but with different values from step to step. Since the solution accounts for a nonuniform initial temperature distribution, the solution for each step can progress from the solution of the previous step. The time period τ varies temporally since ω does vary during the transient. The ratio t_1/τ remains approximately constant if the wheel speed varies slowly with respect to the time period.

Model Equations

The one-dimensional heat conduction equation for the system described and the boundary and initial conditions are, respectively

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{2}$$

at $x = 0$

$$\frac{\partial T}{\partial x} = 0 \tag{3}$$

at $x = L$

$$-k\frac{\partial T}{\partial x} = \begin{cases} q_1 & i\tau < t < (i\tau + t_1) \\ q_2 & (i\tau + t_1) < t < (i+1)\tau \end{cases} \tag{4}$$

at $t = 0$

$$T = T_i(x) \tag{5}$$

where i in Eq. (4) is an integer from zero to some finite value which represents the temporally periodic heat flux at $x = L$, and q_1 and q_2 represent the heating flux at the lands and the cooling flux at the grooves, respectively (Fig. 2). In the coordinate system defined, the heating flux is negative and the

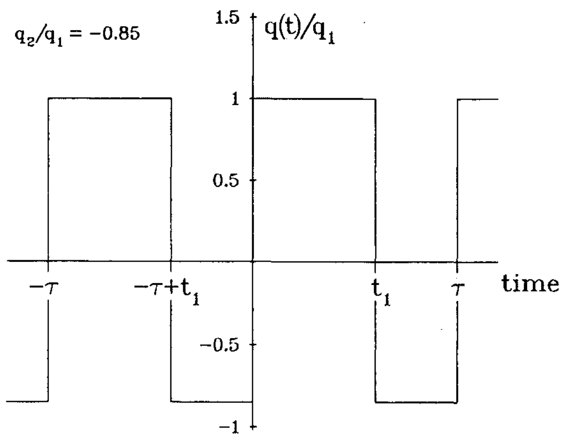

Fig. 2 Schematic of the variation of the time-dependent heat flux boundary condition.

cooling flux is positive. In general, the boundary condition at the interface between the disks is given by

at $x = L$

$$-k\frac{\partial T}{\partial x} = \begin{cases} -k_0\dfrac{\partial T_0}{\partial x} - \mu_s P_N r\omega & i\tau < t < (i\tau + t_1) \\ h(T - T_f) & (i\tau + t_1) < t < (i+1)\tau \end{cases} \tag{6}$$

where 0 denotes the disk for x between L and $2L$. Equation (6) would necessitate solving simultaneously for the temperature distribution in the other disk. For the reaction and driving disk being made of the same material, q_1 can be taken as half of Eq. (1) (i.e., half of the heat generated goes into either disk which assumes the temperature gradient is the same in both disks). Furthermore, if the temporal variation in the disk temperature (T) at $x = L$ is small compared to the fluid and disk temperature difference ($T - T_f$), the heat flux due to the cooling channel can be approximated as a constant. These two assumptions reduce Eq. (6) to Eq. (4), where $q_1 = -0.5\mu_s P_N \omega r$ and $q_2 = h(T - T_f)$.

Solution Method

Equations (2–5) cannot readily be solved using separation of variables due to the time-dependent boundary condition at $x = L$. However, replacing Eq. (4) with a constant heat flux boundary condition

at $x = L$

$$-k\frac{\partial T}{\partial x} = q_1 \tag{7}$$

a solution can be obtained which, when incorporated into Duhamel's method,[9] can be used to solve Eqs. (2–5).

To solve for a constant heat flux at $x = L$, assume a solution of the form

$$T(x, t) = \psi(x, t) + \xi(x) + \beta(t) \tag{8}$$

Substituting into Eqs. (2), (3), (5), and (7), separating variables and solving yields for the temperature distribution

$$T(x, t) = I_0 + \sum_{n=1}^{\infty}\left(I_n e^{-\alpha\lambda_n^2 t}\cos\frac{n\pi x}{L}\right) + \frac{q_1 L}{k}\left\{\frac{1}{6} - \frac{1}{2}\left(\frac{x}{L}\right)^2 - \frac{\alpha t}{L^2} + 2\sum_{n=1}^{\infty}\left[\frac{(-1)^n}{(n\pi)^2}e^{-\alpha\lambda_n^2 t}\cos\frac{n\pi x}{L}\right]\right\} \tag{9}$$

where

$$I_0 = \frac{1}{L}\int_0^L T_i(x)\,\mathrm{d}x \qquad I_n = \frac{2}{L}\int_0^L T_i(x)\cos\frac{n\pi x}{L}\,\mathrm{d}x \tag{10}$$

If the initial temperature distribution is uniform [i.e., $T_i(x) = \bar{T}_i$], Eq. (9) reduces to

$$T(x, t) = \bar{T}_i + \frac{q_1 L}{k}\left\{\frac{1}{6} - \frac{1}{2}\left(\frac{x}{L}\right)^2 - \frac{\alpha t}{L^2} + 2\sum_{n=1}^{\infty}\left[\frac{(-1)^n}{(n\pi)^2}e^{-\alpha\lambda_n^2 t}\cos\frac{n\pi x}{L}\right]\right\} \tag{11}$$

This solution is the same as that given by Carslaw and Jaeger.[10] Equation (9) can be put into a more useable form for this work as

$$Th(x, t) = T(x, t) - I_0 - \sum_{n=1}^{\infty}\left(I_n e^{-\alpha\lambda_n^2 t}\cos\frac{n\pi x}{L}\right) = \frac{q_1 L}{k}\left\{\frac{1}{6} - \frac{1}{2}\left(\frac{x}{L}\right)^2 - \frac{\alpha t}{L^2} + 2\sum_{n=1}^{\infty}\left[\frac{(-1)^n}{(n\pi)^2}e^{-\alpha\lambda_n^2 t}\cos\frac{n\pi x}{L}\right]\right\} \tag{12}$$

where the right side of Eq. (12) is the same as that for an initial condition of zero temperature. For a uniform, initial temperature distribution, $Th(x, t) = T(x, t) - \bar{T}_i$.

To solve for the time-dependent heat flux boundary condition, Eq. (4) is rewritten as

at $x = L$

$$-k\frac{\partial T}{\partial x} = q(t) \tag{13}$$

The function $q(t)$ describing the time-dependent heat flux is neither odd nor even as shown in Fig. 2, and can be represented as a Fourier series as follows:

$$\frac{q(t)}{q_1} = d_0 + \sum_{m=1}^{\infty}\left(d_m\cos\frac{m\pi t}{\tau} + e_m\sin\frac{m\pi t}{\tau}\right) \tag{14}$$

where

$$d_0 = \frac{1}{\tau}\left[t_1 + \frac{q_2}{q_1}(\tau - t_1)\right] \tag{15}$$

$$d_m = \frac{1}{m\pi}\left[\left(\frac{q_2}{q_1} - 1\right)\sin\frac{m\pi(\tau - t_1)}{\tau} + \left(1 - \frac{q_2}{q_1}\right)\sin\frac{m\pi t_1}{\tau}\right] \tag{16}$$

$$e_m = \frac{1}{m\pi}\left\{\left(\frac{q_2}{q_1} - 1\right)\left[\cos\frac{m\pi(\tau - t_1)}{\tau} + \cos\frac{m\pi t_1}{\tau}\right] + \left(1 - \frac{q_2}{q_1}\right)[1 + (-1)^m]\right\} \tag{17}$$

One form of Duhamel's theorem is given as[6]

$$Th'(x, t) = \int_0^t \mathrm{D}(s)\frac{\partial Th(x, t - s)}{\partial t}\,\mathrm{d}s \tag{18}$$

which utilizes the solution due to a stepwise disturbance [Eq. (12)] equal to zero for $t < 0$ and the time-dependent disturbance [Eq. (14)], and where Th' is the temperature distribution due to a time varying boundary condition. Making the substitution of $(t - s)$ for t in Eq. (12), and taking the derivative with respect to t yields

$$\frac{\partial Th(x, t - s)}{\partial t} = \frac{q_1 \alpha}{kL}\left\{-1 - 2\sum_{n=1}^{\infty}\left[(-1)^n \exp[-\alpha\lambda_n^2(t - s)]\cos\frac{n\pi x}{L}\right]\right\} \tag{19}$$

with D(s) being given by

$$D(s) = d_0 + \sum_{m=1}^{\infty}\left(d_m \cos\frac{m\pi s}{\tau} + e_m \sin\frac{m\pi s}{\tau}\right) \tag{20}$$

Substitution of Eqs. (19) and (20) into Eq. (18) gives the following result:

$$\frac{Th'(x, t)}{(q_1 L/k)} = -\frac{\alpha}{L^2}\sum_{m=1}^{\infty}\left\{\frac{\tau}{m\pi}\left[d_m \sin\frac{m\pi t}{\tau} - e_m\left(\cos\frac{m\pi t}{\tau} - 1\right)\right]\right\} + d_0\left\{\frac{1}{6} - \frac{1}{2}\left(\frac{x}{L}\right)^2 - \frac{\alpha t}{L^2} + 2\sum_{n=1}^{\infty}\left[\frac{(-1)^n}{(n\pi)^2} e^{-\alpha\lambda_n^2 t}\cos\frac{n\pi x}{L}\right]\right\}$$
$$- 2\frac{\alpha}{L^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{\frac{(-1)^n}{(\alpha\lambda_n^2)^2 + \left(\dfrac{m\pi}{\tau}\right)^2}\cos\frac{n\pi x}{L} \times \left[\left(\frac{m\pi}{\tau}e_m - \alpha\lambda_n^2 d_m\right)e^{-\alpha\lambda_n^2 t} + \alpha\lambda_n^2\left(d_m\cos\frac{m\pi t}{\tau} + e_m \sin\frac{m\pi t}{\tau}\right) + \frac{m\pi}{\tau}\left(d_m \sin\frac{m\pi t}{\tau} - e_m\cos\frac{m\pi t}{\tau}\right)\right]\right\} \tag{21}$$

where Eq. (21) is the solution to Eqs. (2–5) and

$$Th'(x, t) = T'(x, t) - I_0 - \sum_{n=1}^{\infty}\left(I_n e^{-\alpha\lambda_n^2 t}\cos\frac{n\pi x}{L}\right) = T'(x, t) - T_i(x) + \sum_{n=1}^{\infty}\left[I_n(1 - e^{-\alpha\lambda_n^2 t})\cos\frac{n\pi x}{L}\right] \tag{22}$$

The following substitution determined from Eq. (9) at $t = 0$ was used in Eq. (21):

$$\frac{1}{6} - \frac{1}{2}\left(\frac{x}{L}\right)^2 = -2\sum_{n=1}^{\infty}\left[\frac{(-1)^n}{(n\pi)^2}\cos\frac{n\pi x}{L}\right] \tag{23}$$

Numerical evaluation of Eq. (21) is straightforward on a digital computer. Typical solutions in this work were determined using 50 terms in each series (i.e., for m and n). Doubling the number of terms in either or both series resulted in a change in the temperature distribution of less than 2% for the conditions in this work.

Results

As necessary, Eq. (21) reduces to Eq. (11) when q_2 is equal to q_1 (which yields $d_0 = 1$ and $d_m = e_m = 0$), and $T_i(x) = \bar{T}_i$. Some of the salient features of the process described herein can be examined by setting $-q_1 = q_2$ and $2t_1 = \tau$ (the heating and cooling fluxes are opposite and equal and of equal time duration). Figure 3 shows the first cycle of the transient. For the disk thickness and thermal diffusivity specified, the heat flux does not penetrate the depth of the disk. Even though the time period for the heating and cooling cycle are equal, the temperature change above and below the initial temperature is not equal, which results in some offset of the average temperature of the disk. This trend can be explained further by a plot of the surface temperature at $x = L$ with time as given in Fig. 4. The area under the curve below $\theta = 0$ is greater, which indicates the rest of the disk temperature will shift due to the heat conducted from the disk at $x = L$ to $x = 0$ (i.e., the process begins with one or the other of the fluxes and correspondingly is biased in that way). The surface temperature becomes quasisteady after about five cycles. The quasisteady temperature distribution for the heating and cooling part of the cycle are a mirror image of each other as would be expected for equal absolute values of the heating and cooling fluxes and time periods.

Figure 5 is a plot of the temperature distribution in the disk for the 50 first cycle with α/L^2 10 times larger than that of Fig. 3. The heat flux in this case does penetrate the disk and the temperature at $x = 0$ oscillates with every cycle. Again, Fig. 5a is the mirror image of Fig. 5b (quasisteady state). The effects of reducing α/L^2 by a factor of 10 (compared to Fig. 3) is not shown for brevity. The heat flux penetration is less, and the disk at $x = 0$ does not respond to the temperature oscillations at $x = L$ due to the smaller thermal diffusivity (or larger disk thickness).

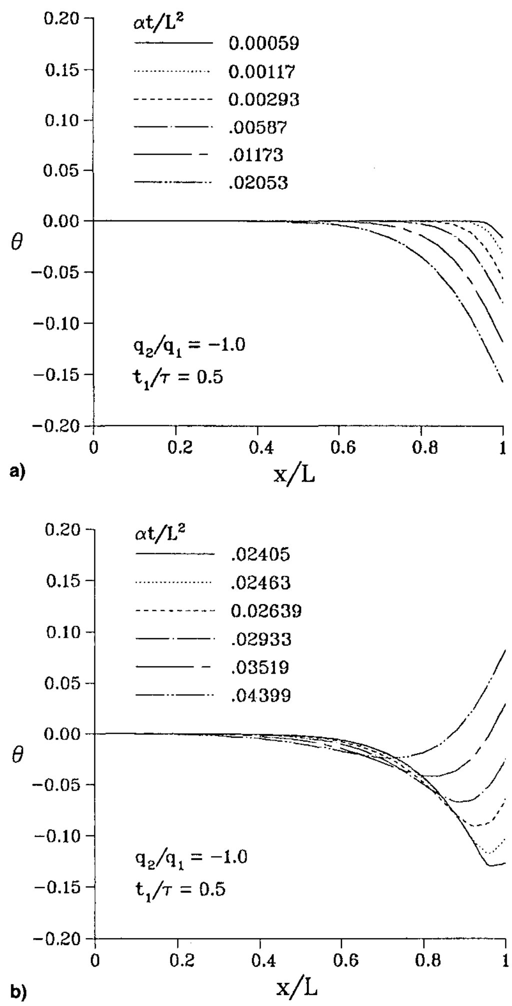

Fig. 3 Plot of first cycle of transient: a) heating flux and b) cooling flux ($\alpha/L^2 = 0.0117$ s^{-1}).

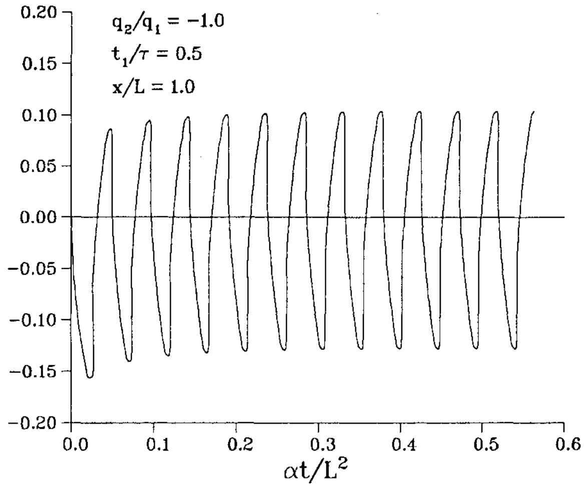

Fig. 4 Time history of surface temperature at $x = L$ ($\alpha/L^2 = 0.0117$ s^{-1}).

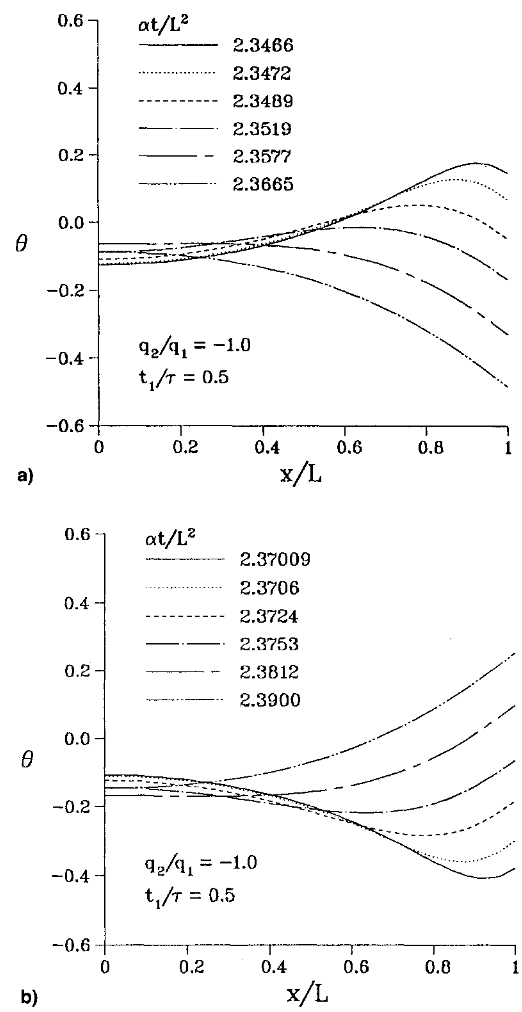

Fig. 5 Plot of 51 cycle of transient: a) heating flux and b) cooling flux ($\alpha/L^2 = 0.117$ s^{-1}).

A more typical situation is with the duration of the cooling flux being less than that of the heating flux, and with the fluxes not being of equal absolute value. To isolate the trends, only one of these will be changed. For example, the heat fluxes will be kept equal and opposite, but the time duration of the cooling flux of the cycle will be one-fourth the time duration of the heating flux. The quasisteady state temperature distribution is given in Fig. 6. The temperature distribution is similar to that for a steady heat flux boundary condition,[10] but with a small perturbation superimposed on the distribution at the end corresponding to $x = L$. The penetration depth of the cooling flux is smaller than that due to the heating flux since the time duration is shorter. A time history of the temperature at $x = L$ (surface where heat flux is applied) is given in Fig. 7. The dimensionless temperature profile reaches a quasisteady state after several cycles, although the actual temperature profile continually increases with time. A similar trend would be expected for equal time durations of the heating and cooling part of the cycle and unequal absolute values of the heat fluxes. This quasisteady state occurs at later times as q_2/q_1 and t_1/τ differ further from one.

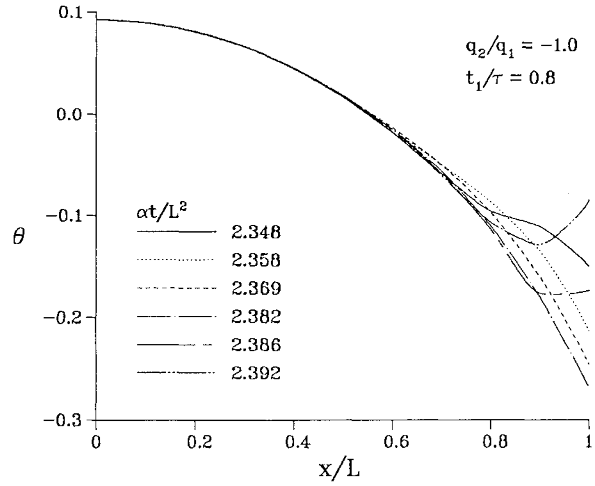

Fig. 6 Plot of quasisteady temperature for $2t_1$, not equal to τ ($\alpha/L^2 = 0.0117$ s^{-1}).

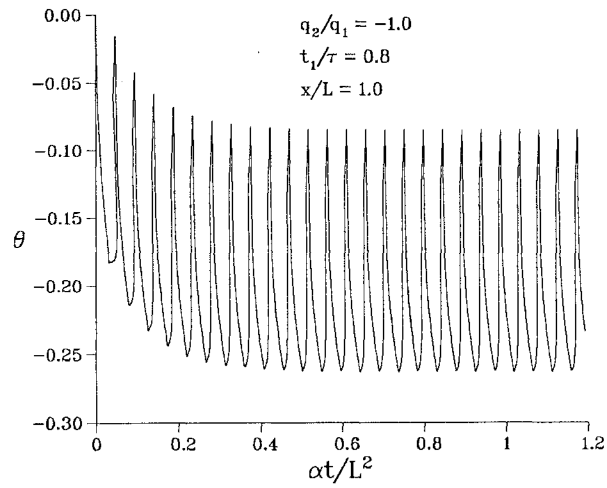

Fig. 7 Time history of surface temperature at $x = L$ ($\alpha/L^2 = 0.0117$ s^{-1}).

All of the preceding examples were for a uniform, initial temperature distribution. The effects of a nonuniform initial temperature distribution are twofold. At early times, the nonuniform distribution of energy in the plate is reapportioned with respect to the heat flux at the boundary and the thermal properties of the plate. At later times, the temperature distribution is biased by the average temperature of the initial temperature distribution, I_0 [i.e., the initial distribution is smoothed out such that at later times the solution appears to have been started from a uniform temperature distribution equal to the average of $T_i(x)$]. When $T_i(x)$ is not constant, the integrals in I_0 and I_n are required. Numerical integration is plausible, but inaccuracies were present depending on the number of integration points and terms in the series being used. A tractable solution was obtained by curve-fitting the temperature distribution to some function which can be integrated analytically. In some previous design work, a second-order polynomial was used satisfactorily.

As a concrete example, consider a multiplate clutch which transmits a torque of 0.306 N-m/cm^2 (initial power dissipation is 1 hp/in.^{2}). The driving and reaction wheels are made of the same material (carbon/carbon friction material, $\alpha = 9.497 \times 10^{-7}$ m^2/s and $k = 15.5$ W/mK), and correspondingly, the heating flux is half the heat generation at the interface. The initial wheel speed difference (ω_i) is 377 rad/s (3600 rpm), and the disk half-thickness is 0.191 cm. The mean radius is 7.62 cm and there are 15 radial grooves 1.27-cm wide. The decrease in wheel speed is linear over a 5-s engagement time, and correspondingly, τ and t_1 vary linearly from an initial value to infinity, and q_1 varies linearly from an initial value to zero. Since q_1 is not constant with time, the boundary heat flux is not periodic over the time period τ and cannot be represented as a Fourier series. For modeling, the engagement time is broken into five equal time steps, and over each step q_1, τ, and t_1 are taken as constant equal to the average value for that time step. For example, in the second time step, $q_1 = 0.7\omega_i Y$ and $\tau = \Delta\phi/(0.7\omega_i)$. Assume the initial temperature is 82.1°C (inlet oil temperature), and the cooling flux is constant at 20% of the initial heating flux (the actual cooling flux varies somewhat due to the disk temperature changing). These constraints result in $t_1/\tau = 0.6$, and from initial conditions in $q_2/q_{1i} = -0.2$. These conditions are referred to as the base case.

Figure 8 illustrates the spatial temperature distribution at the end of each 1-s time step. Since the heating flux decreases with time, a maximum temperature is reached after which the cooling flux is larger than the heating flux and the disk decreases in temperature (time period of heating is greater than that of cooling during the entire engagement). There is some temperature gradient across the disk even though the disk is taken as adiabatic (symmetry condition) at $x = 0$, due to the cyclic heating and cooling. About 900 heating and cooling cycles have occurred during the first 1-s time step (based on the average wheel speed difference for the first second). Therefore, the shape of the temperature distribution has been established early in the time step (as discussed previously). The temperature gradient across the disk changes because the cooling and heating ratio changes. The temporal variation of the heat flux is not modeled exactly, but the trend is correct. Further refinement can be achieved by dividing the engagement time into smaller steps, accounting for the change in q_2 by using the disk temperature to improve the value of q_2 at each time step, etc.

If the time period ratio t_1/τ is decreased to 0.5 from 0.6 (time period of cooling and heating is increased and decreased, respectively), the maximum temperature at 4 s is decreased by about 29% (based on the initial and maximum, at 4 s, disk temperature difference of the base case). The overall trend with time is the same. The ratio t_1/τ can also be viewed as the area ratio of the land to total disk area. This is helpful in evaluating complicated channel geometries. If the coolant flow rate is decreased such that $q_2/q_1 = -0.1$, the maximum temperature (at 4 s) increases by about 15% due to the decreased cooling. Similarly, the maximum temperature (at 4 s) is decreased about 24 and 57% due to increasing, respectively, the disk half-thickness by a third, and changing the disk material to a low grade steel (α and k increase by a factor of 1.6 and 3.8, respectively). Equation (2) can be rearranged as $\partial T/\partial t = -(\alpha/k)\, \partial q/\partial x$. Since the boundary heat flux is the same for either disk material considered and the increase in the thermal conductivity is greater than that of the thermal diffusivity, the maximum disk temperature decreased. These trends are as expected, but do quantify the various design parameters. For example, using oil to cool the clutch would place a limit on the maximum disk temperature to prevent thermal degradation of the oil (occurs around 175–205°C) and some action (thicker disks, different material, higher oil flow rate, etc.) would be required to maintain this upper limit on the disk temperature.

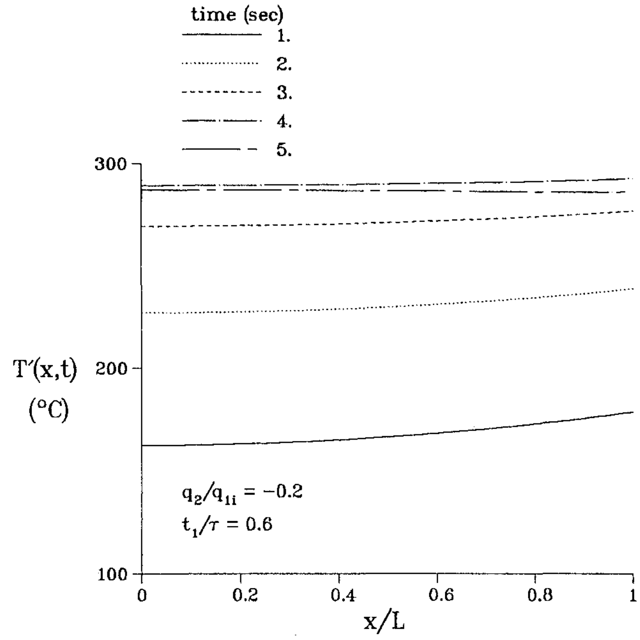

Fig. 8 Temperature distribution history for a carbon/carbon friction material disk and a 5-s engagement time.

The fraction of the heat generation at the interface which enters each of the two disks in contact must be determined if the disk materials are different. Consider two disks with $x = 0$ at the interface between the disks. The disk on the left extends from $x = 0$ to $x = -L_a$, and the disk on the right from $x = 0$ to $x = L_b$. At the interface ($x = 0$), the heat flux boundary condition is some fraction of the heat generation given as fq for the left-hand disk (subscript a) and $(1 - f)q$ for the right-hand disk (subscript b), where f is between 0 and 1. The heat flux is constant with time for this portion of the work and the surfaces at $x = -L_a$ and $x = L_b$ are adiabatic. The remaining constraint is that the surface temperature of each disk must be equal at $x = 0$ (assuming negligible contact resistance). Solving the one-dimensional energy equation for each disk [the same method used to solve Eqs. (2), (3), (5), and (7)], setting them equal at $x = 0$ and solving for f yields

$$f = \left[1 + \frac{k_b L_a}{k_a L_b} \, \frac{\dfrac{\alpha_a t}{L_a^2} + \dfrac{1}{3} - 2\displaystyle\sum_{n=1}^{\infty} \left\{ \frac{1}{(n\pi)^2} \exp\left[-\left(\frac{\alpha_a t}{L_a^2} \right) (n\pi)^2 \right] \right\}}{\dfrac{\alpha_b t}{L_b^2} + \dfrac{1}{3} - 2\displaystyle\sum_{n=1}^{\infty} \left\{ \frac{1}{(n\pi)^2} \exp\left[-\left(\frac{\alpha_b t}{L_b^2} \right) (n\pi)^2 \right] \right\}} \right] \tag{24}$$

The parameter f is a function of time, but if the properties of disk a and b are equal, f is a constant equal to 0.5. In the limit as t approaches 0, f reduces to

$$f = \left(1 + \frac{k_b L_a}{k_a L_b} \right)^{-1} \tag{25}$$

At time just after zero, the heat must be conducted into the disk before any temperature increase can occur, and as such depends on the thermal conductivity. In the limit as t approaches infinity, f reduces to

$$f = \left[1 + \frac{(\rho c L)_b}{(\rho c L)_a}\right]^{-1} \qquad (26)$$

Physically, once the temperature profile in the disk is established, any further energy added to the disk only alters the level of the temperature profile, not the shape of the profile. If $(k_b L_a)/(k_a L_b)$ and $(\rho c L)_b/(\rho c L)_a$ are equal, f is constant with time and equal to the value given by either Eq. (25) or (26). Otherwise, Eqs. (25) and (26) are the bounds for f during any given transient. Typically for metal disks, the steady state value of f is achieved quickly.

In general, the physical situation Eq. (24) is based on (constant total heat flux into both disks) does not correspond to the physical situation Eq. (21) is based on (periodic heat flux). If either flux is smaller or of a shorter time duration than the other flux, the temperature profile behaves similarly to that shown in Fig. 6 (i.e., a small perturbation on a quasisteady temperature distribution. These conditions are better approximated by the physical situation used in developing Eq. (24), and some time-weighted average of f bounded by Eqs. (25) and (26) can be used. This yields a suitable approximation for the temperature profile at the end of the transient, but intermediate values are less accurate (total energy input for a given time period is modeled). If the total time for the transient is such that the larger part of the transient occurs under a quasisteady temperature distribution, the steady state value of f [Eq. (26)] can be used as an approximation.

Conclusions

A method to determine the one-dimensional, transient temperature distribution due to a temporally periodic heat flux at one boundary has been presented. The periodic heat flux corresponds to the heat generation due to two disks sliding on each other and the oil groove (cooling flux) passing an arbitrary point on the ungrooved disk. For two disks in sliding contact of different materials without cooling, a measure of the fraction of the heat generated at the interface which goes into each disk was determined.

These solutions can be used together in determining maximum material temperature limits for a wide variety of periodically heated and cooled disks or plates (e.g., transmission clutches and brakes). Furthermore, the maximum and minimum cyclic temperatures are useful in determining high-cycle fatigue due to thermal stresses. As expected, increasing (reducing) the heating (cooling) flux, increasing (reducing) the time period of the heating (cooling) flux, decreasing the disk thickness or increasing the disk thermal diffusivity increases the disk temperature during the engagement time.

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